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Lunar Landing and Long-Range Earth Re-Entry Guidance by Application of Perturbation Theory

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A guidance scheme based on linear perturbation theory has been investigated. An improved capability has been achieved by the proper choice of independent variable and by appropriate weighting of the guidance gains computed by linear theory. The capability of this scheme applied to the descent-to-hover phase of lunar landing is demonstrated for two different types of nominal trajectory: a constant-thrust gravity turn maneuver, and a constant-thrust, constant-pitch-rate maneuver. To demonstrate the performance of this type of guidance scheme for atmosphere entry, it has been applied to the guidance of a vehicle entering the earth's atmosphere at parabolic velocity. Its capability is evaluated for entries from abort conditions, as well as for entries within the normal entry corridor, and effects of variations in life-drag ratio and atmospheric density are investigated. It is shown that for both lunar landing and atmosphere entry this guidance system, which uses a single nominal trajectory, and therefore requires minimum storage capacity, permits guidance to a selected landing site from a wide range of initial conditions.

Nomenclature

A = aerodynamic acceleration, g units
C_D = drag coefficient, dimensionless
$F_{\alpha\eta}$ = linear theory gain for the α state variable used to determine the magnitude of the control variable η, dimensions of η/α
g = surface gravity, ft/sec²
h = altitude, ft
I_{sp} = specific impulse, sec
$K_A, K_{\dot{x}}$ = empirical dimensionless weighting factors
L/D = lift-drag ratio, dimensionless
m = mass, lb-sec²/ft
r = earth radius plus altitude, $r_e + h$, ft
S = vehicle reference area, ft²
t = time, sec
T = thrust, lb
V = total velocity, fps
V_c = characteristic velocity, $gI_{sp}\ln(m_i/m_f)$, fps
W = earth weight, lb
x = downrange, ft or statute miles
x_{TG} = range to go, $x_f - x$, ft or statute miles
$(\dot{\ })$ = derivative with respect to independent variable
γ = flight-path angle, deg
$\delta(\ )$ = difference between actual and reference value of any quantity, $(\ ) - (\ )_r$
θ = thrust orientation, positive upward, deg
$\lambda_{\alpha\eta}$ $(\partial\eta/\partial\alpha)$ adjoint variable

Subscripts

i = initial
f = final
r = reference or nominal

Introduction

IN the field of guidance of aerospace vehicles, the concept of guidance about a nominal or reference trajectory has received considerable attention. It has been investigated for use in the midcourse phase of the lunar mission, and for vehicles re-entering the earth's atmosphere (e.g., Refs. 1–5). The mathematical basis for this concept is perturbation theory,[6, 7] that is, the analysis of conditions in a limited neighborhood of a nominal trajectory. Nonlinear systems may be handled by the theory, because in the neighborhood of a known trajectory they can be described in terms of linear differential equations with varying coefficients. However, it is the restriction to the neighborhood of a known trajectory which forms the primary limitation on the usefulness of the theory, particularly in the case of atmospheric re-entry. The various proposed methods employing this concept for re-entry have, in general, range capability of 6000 miles or less. Theoretically, the proposal of Ref. 5 to store multiple nominal trajectories and associated feedback gains should permit guidance to any range, but the increased storage capacity required makes a search for a simpler approach desirable. The purpose of this paper is to show that by the proper choice of independent variable and use of empirical weighting factors it is possible greatly to increase the guidance capability of a linear perturbation scheme. This capability is achieved without increase in information storage requirements.

Theory

Development of Control Equation

In this section we will derive the basic equation used in linear perturbation guidance. Somewhat similar developments may be found in the literature (e.g., Ref. 7). Consider the set of nonlinear differential equations

$$\dot{x}_m = F_m(x_n, u_p, v) \tag{1}$$

where

$1 \leq n \leq M$
$F = M$ known functions
$x = M$ state variables
$u = P$ external force variables
v = independent variable (such as time, velocity, etc.)

Expanding Eq. (1) in a Taylor series about some desired nominal or reference trajectory and retaining terms to first order only gives

$$\delta\dot{x} - \sum_M a_{mn}\delta x_n = \sum_P b_{mp}\delta u_p \tag{2}$$

This is a set of M linear differential equations with varying

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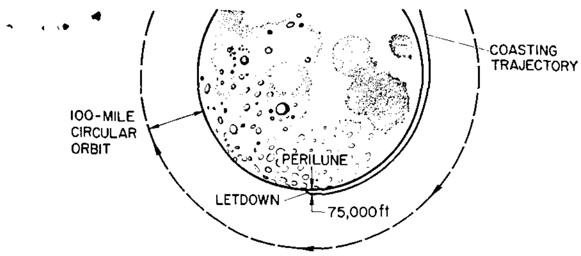


Fig. 1 Lunar landing approach.

coefficients $a_{mn}(v)$ and $b_{mp}(v)$, the solution of which describes the motion about the reference trajectory, where

$$\delta x_n(v) = x_n(v) - x_{n_r}(v)$$

$$a_{mn}(v) = \left(\frac{\partial F_m}{\partial x_n}\right)_r (v)$$

$$b_{mp}(v) = \left(\frac{\partial F_m}{\partial u_p}\right)_r (v)$$

The set of equations adjoint to Eq. (2) is defined by

$$\dot{\lambda}_m + \sum_M a_{nm}\lambda_n = 0 \tag{3}$$

Multiplying Eq. (2) by λ_m, Eq. (3) by δx_m, summing over M and integrating over the interval v to $v_f(v_i \leq v \leq v_f)$ gives

$$\sum_M \lambda_m \delta x_m \Big|_{v_f} = \sum_M \lambda_m \delta x_m \Big|_v + \int_v^{v_f} \sum_M \sum_P b_{mp}\lambda_m \delta u_p dv_1 \tag{4}$$

This is the basic equation for control about a reference condition and was called by Bliss[6] the fundamental formula. Equation (4) may be particularized by identifying the single sum at $v = v_f$ with the state variable x_q, which it is desired to control $(1 \leq q \leq M)$. Thus, identify

$$\sum_M \lambda_m \delta x_m \Big|_{v_f} = \delta x_q \Big|_{v_f} \tag{5}$$

Then

$$\lambda_m \Big|_{v_f} = \frac{\partial x_q}{\partial x_m}\Big|_{v_f} \tag{6}$$

To indicate the proper partial derivative, the following notation has been introduced in the literature. Equation (6) is written

$$\lambda_{x_m}{}^{x_q}(v_f) = \frac{\partial x_q}{\partial x_m}\Big|_{v_f} \tag{7}$$

Equation (7) defines the boundary conditions necessary for the solution $\lambda_{x_m}{}^{x_q}(v)$ of Eq. (3). Equation (4) may now be written

$$\delta x_q(v_f) = \sum_M \lambda_{x_m}{}^{x_q}(v)\delta x_m(v) + \int_v^{v_f} \sum_M \sum_P b_{mp}\lambda_{x_m}{}^{x_q}\delta u_p dv_1 \tag{8}$$

Equation (8) is the basic equation by means of which an estimate can be made of the first-order change δx_q of the state variable x_q from its reference value at the final condition v_f, due to 1) a change δx_m of any state variable x_m from its reference value at a prior condition v, and 2) a change δu_p of any external force variable u_p from its reference value during the interval v to v_f.

For simplicity, consider u_p to be control variables, and assume that the number q of state variables it is desired to control is equal to the number P of control variables. Then, given a desired final value $\delta x_q(v_f)$ and given certain departures $\delta x_m(v)$, there is an infinity of control variable functions which will accomplish the desired final value. In particular, there is a constant value δu_p over the interval $v \leq v_1 \leq v_f$ which will accomplish the desired final value, and, with the notation,

$$Iu_p{}^{x_q}(v) = \int_v^{v_f} \sum_M b_{mp}\lambda_{x_m}{}^{x_q} dv_1 \tag{9}$$

Eq. (8) may be written

$$\delta x_q(v_f) = \sum_M \lambda_{x_m}{}^{x_q}(v)\delta x_m(v) + \sum_P I_{u_p}(v)\delta u_p \tag{10}$$

Solution of Eq. (10) for the control variables u_p then gives

$$u_p(v) = u_{p_r}(v) + \sum_M F_{x_m}{}^{u_p}(v)\delta x_m(v) \tag{11}$$

Equation (11) is applicable to a complete three-dimensional analysis. All the guidance results obtained in this paper are two dimensional. It can be shown that to first order, these results are valid for three-dimensional applications.

Insofar as the theory is concerned, the particular set of state and independent variables chosen is completely arbitrary. There are practical considerations for using a state variable as independent variable rather than time, since this reduces M in Eq. (11) by 1, thus simplifying the guidance through reduced information storage requirements. In the present investigation the state variables chosen were somewhat arbitrary, but total velocity instead of time was used as independent variable because of the simplification just noted, and because it appears that it has additional advantages as well. The most significant advantage is that the neighborhood of the nominal trajectory appears to be larger in terms of velocity, or, perhaps more correctly, the excursions of the state variables on the actual trajectories relative to those on the nominal trajectory generally are smaller when compared on the basis of velocity. Obviously, the advantage of using the independent variable for which the δx_m of Eq. (11) are minimized is that less violence must be done to the linear theory to make it operate over the range of conditions desired.

Equation (11) defines a terminal control system in that the system makes no attempt to eliminate present errors, but instead acts to prevent the propagation of present errors of all variables into errors of the controlled variables at the final condition $v = v_f$. As formulated, the system defined by Eq. (11) attempts to use minimum control excursion for a maximum length of time. If the information possessed by the system is correct in the sense that all pertinent variables have been accounted for, and if the system is in the neighborhood of the nominal trajectory where linearization of the equations is valid, Eq. (11) will command a control increment just sufficient to achieve the desired result if the increment is maintained to the final condition. Another formulation which has been used is to command the maximum available control excursion for a minimum amount of time.[3] However, during the earth re-entry portion of the present investigation, it was found that (when attempting to

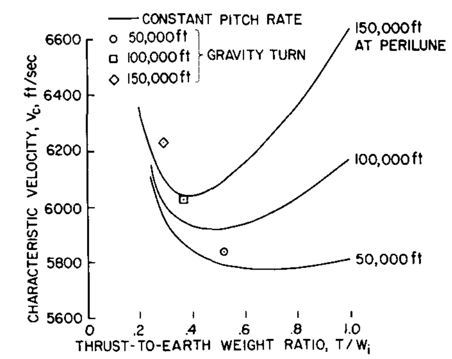


Fig. 2 Fuel required for gravity turn and constant-pitch-rate maneuver.

operate outside the region wherein linearization is valid) this type of command tended to cause erroneous trajectory excursions from which it was later impossible to recover, and so was not satisfactory. The form of control finally used was intermediate to these two extremes; the guidance gains were adjusted through the use of empirically determined weighting functions, as will be described subsequently. By these two means, the use of velocity as independent variable and empirical weighting of the guidance gains, it was possible to extend greatly the operating range of the basic linear theory.

Lunar Landing

The guidance scheme just described will now be applied to the descent-to-hover phase of lunar landing. The main features of the descent from orbit to the lunar surface are indicated in Fig. 1. A 100-mile circular orbit was assumed, with the gross descent accomplished by means of a Hohmann transfer orbit whose perilune determined the initial conditions for that portion of the descent considered here—a guided letdown to an altitude of less than 1000 ft.

Two maneuvers previously considered in the literature[8,9] were chosen as reference trajectories, the gravity turn, and the constant-pitch-rate maneuvers. The fuel required, in terms of the characteristic velocity, is shown in Fig. 2 for the two maneuvers as it is affected by initial or perilune altitude and thrust level. Any desired value of thrust may be used for the constant-pitch-rate maneuver for a given initial altitude, in contrast to the single value of thrust necessary for the gravity turn. Both maneuvers require a large increase in fuel with increase in initial altitude. As a compromise between fuel requirements and avoiding the mountainous lunar surface, an initial altitude of 75,000 ft was chosen. This prescribed a $T/W_i = 0.42$ for the gravity turn. The thrust ratio for the constant-pitch-rate maneuver was chosen to be 0.56, the optimum value for this altitude.

The characteristics of the resultant reference trajectories are shown in Fig. 3 in terms of the state variables chosen for use in the control equation; altitude h, flight-path angle γ, and range x. It is desired to control the final values of two quantities, range and altitude. The two control variables are thrust T and thrust orientation θ. Then the two control equations from Eq. (11) are

$$T(V) = T_r + F_h^T(V)\delta h(V) + F_\gamma^T(V)\delta\gamma(V) - F_x^T(V)\delta x_{TG}(V) \qquad (12)$$
$$\theta(V) = \theta_r(V) + F_h^\theta(V)\delta h(V) + F_\gamma^\theta(V)\delta\gamma(V) + F_x^\theta(V)\delta x_{TG}(V)$$

The guidance gains associated with Eq. (12) are presented in Fig. 4. The gains associated with flight-path angle remain finite over the velocity range; however, all other gains for both reference trajectories have singularities at zero velocity. Since this investigation was performed on an analog computer, rather severe storage limitations were

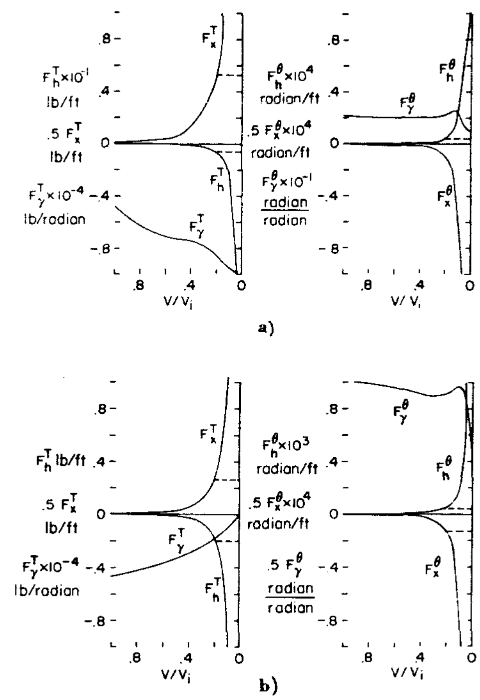


Fig. 4 Linear theory guidance gains: a) constant pitch rate, b) gravity turn.

necessarily imposed, a circumstance, however, which conforms with the original intent of developing a guidance system with minimum information storage requirements. The dashed lines in Fig. 4 indicate the maximum values of the gains actually used in the investigation.

The guidance capability using the constant-pitch-rate reference trajectory is summarized in Fig. 5 in terms of the corridor of initial altitude and range limits from which it is possible to guide to a target area 1000 ft in altitude and 10,000 ft in range, the center of which is located 770,000 ft downrange, the range for this particular reference trajectory. Changes in initial range and altitude were accompanied by the appropriate initial velocity and flight-path angle changes corresponding to the Hohmann trajectory passing through that point.

Two corridors are shown, the smaller one corresponding to the use of the guidance gains shown in Fig. 4. Multiplying the gain F_h^θ by a factor of 2 more than doubled the size of the corridor. Further increases in guidance capability were found to be possible by the same means, but characteristics such as increased fuel requirements, excessive angular rates, and other factors made the results unsatisfactory.

Also shown in Fig. 5 are the fuel requirements for a perilune altitude of 75,000 ft. Only a moderate fuel increase occurs

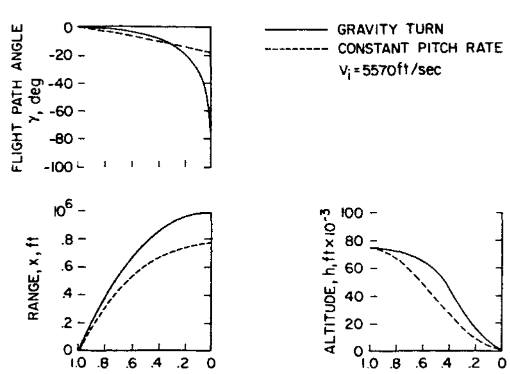


Fig. 3 Reference trajectory state variables.

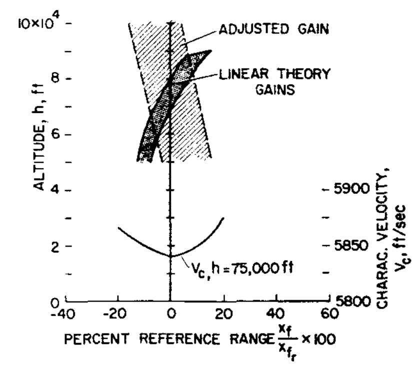


Fig. 5 Guidance capability: constant-pitch-rate reference trajectory.

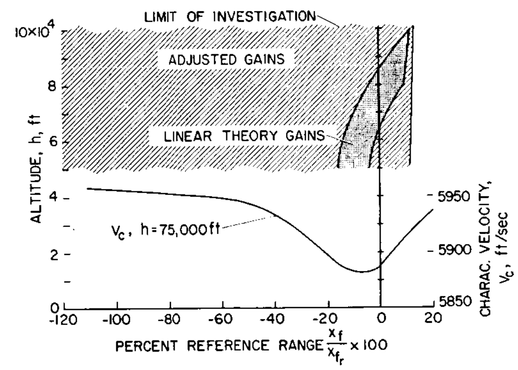


Fig. 6 Guidance capability: gravity turn reference trajectory.

for initial range errors. The curve shown results from use of the linear theory gains; however, the fuel requirements due to using the adjusted gain is not significantly different.

Figure 6 shows the guidance capability obtained with the gravity turn reference trajectory. Again, the smaller corridor was obtained using the guidance gains of Fig. 4. By a moderate adjustment of the guidance gains, namely, the use of $1.5F_{\dot{h}}^{\theta}$ and $0.85F_x^{\theta}$, the extreme increase in the guidance capability shown in the figure was obtained. The fuel requirement is shown in the lower part of the figure for the nominal altitude of 75,000 ft. A 100% range error requires a fuel increase of about 70 fps, which is equivalent to about 14 sec of hover time. As noted, the only limit actually defined for this case is the downrange limit; the edges of the cross-hatched area correspond to the range of conditions investigated. Thus, the actual amount of weighing applied to the linear gains was somewhat arbitrary. No attempt was made to define the other limits, because possibly there are important constraints such as target visibility which have not been considered in this investigation and which in an actual application would sufficiently define the problem such that an optimum distribution of weighing can be determined. The important result shown here is that the rather limited capability of the basic linear guidance scheme can be greatly extended by very simple means.

Earth Re-Entry Guidance

In the investigation of guidance for atmosphere re-entry at parabolic speed, a vehicle with a maximum $L/D = 0.4$ and $W/C_DS = 50$ was chosen. The nominal atmosphere used was the 1959 ARDC model.

The characteristics of the nominal trajectory chosen are shown in Fig. 7. Several factors were considered in choosing this trajectory. One factor was the easing of the restrictions upon the time of return from a lunar mission to a single earth site by seeking ranges up to one-half the earth's circumference. A 6000-mile nominal range was chosen since it is approximately in the center of the desired range envelope. A high-skip type of trajectory was chosen because it imposes low total heat loads, and because the final range is less sensitive to state variable errors than it is for trajectories which have relatively low skip altitudes. Although not of concern in the present investigation, these considerations are of great practical significance in the design of the heat shield, and the backup and monitoring system for the primary guidance.

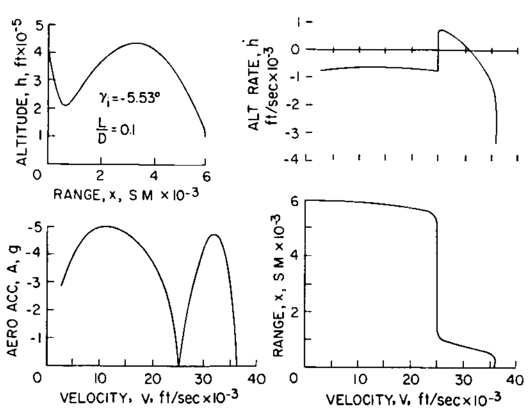


Fig. 7 Reference trajectory state variables.

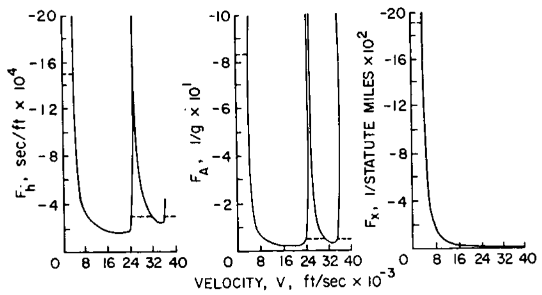


Fig. 8 Linear theory guidance gains.

The nominal L/D for this trajectory is equal to 0.1. Figure 7 shows the characteristics of the reference trajectory in terms of the state variables chosen for use in the control equation: altitude rate $\dot{h}$, aerodynamic acceleration A, and range x. It is desired to control the single quantity, range, and by means of the control variable L/D; then the control Eq. (11) is

$$L/D(V) = (L/D)_r + F_{\dot{h}}(V)\delta\dot{h}(V) + F_A(V)\delta A(V) - F_x(V)\delta x_{TG}(V) \quad (13)$$

The guidance gains associated with Eq. (13) are shown in Fig. 8. These gains, determined by means of the linear theory, did not define a guidance scheme capable of handling the nonlinearities resulting from the large departures from the reference trajectory desired, even when account was taken of the multivalued nature of velocity evident in Figs. 7 and 8. The gains actually used in the results to be presented are indicated by the dashed lines. Associated with these gains are the empirically determined weighting factors shown in Fig. 9 which enable the guidance system to operate over virtually the full range of vehicle capability. The combination shown is not unique; other combinations also permit full guidance capability, a fact which will allow for optimization studies. The final form of the control equation is now

$$L/D(V,x_f) = (L/D)_r + K_{\dot{h}}(V,x_f)F_{\dot{h}}(V)\delta\dot{h}(V) + K_A(V,x_f)F_A(V)\delta A(V) - F_x(V)\delta x_{TG}(V) \quad (14)$$

This equation with the modified linear theory gains and the weighting factors just described were used to obtain all the results to be presented subsequently.

Figure 10 shows a typical guided trajectory for an entry angle corresponding to one of the extremes permitted by the

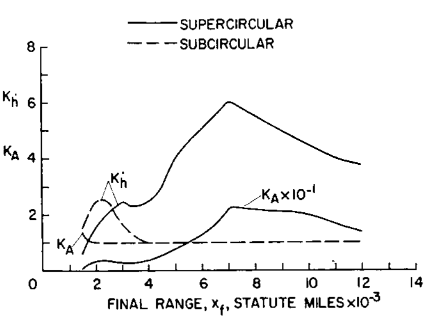


Fig. 9 Empirical weighting factors.

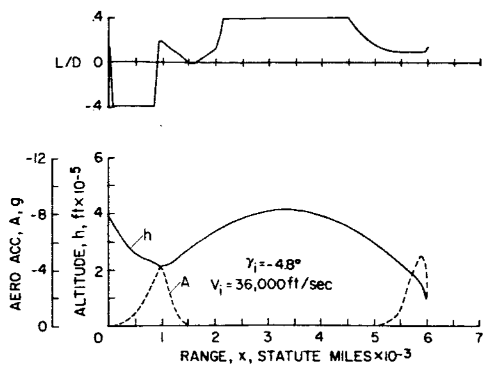


Fig. 10 Typical shallow entry guided trajectory.

vehicle's capability, the uncontrolled skip boundary. The uncontrolled skip boundary is defined as the shallowest re-entry angle at which the vehicle can acquire sufficient aerodynamic force to control the subsequent trajectory to the shortest desired range. For the vehicle considered in this study the boundary is $\gamma_i = -4.7°$. Another boundary defined by the vehicle's capability is the maximum acceleration boundary which is defined as the steepest re-entry angle for which the acceleration will not exceed the maximum desired. For the 10-*g* limit chosen for this study, the boundary is $\gamma_i = -7.3°$.

These boundaries are shown in Fig. 11; for ranges greater than approximately 6300 miles another vehicle capability boundary is defined by the maximum range possible at a given re-entry angle. Also shown in Fig. 11 are the data points indicating guided trajectories calculated to delineate the guidance capability. It can be seen that the guidance system is capable of operating over virtually the entire corridor defined by the vehicle itself.

An item of significant interest in the evaluation of a guidance system is its ability to handle off-design conditions. Three types of off-design conditions were considered in this study: 1) re-entry from abort conditions, 2) variations in the vehicle L/D, and 3) atmospheric variations. The abort conditions considered were re-entries from circular orbit and from a velocity of 32,000 fps. The two re-entries shown in Fig. 12a were initiated from a circular orbit at an altitude of 600,000 ft. The range traversed from the time of leaving orbit altitude until an altitude of 400,000 ft was reached was 4700 miles greater for the $\gamma_i = -0.41°$ entry than for the $\gamma_i = -1.59°$ entry. Comparison of this range increment with that possible through guidance as shown in Fig. 12a indicates that the thrust applied in orbit to initiate re-entry must be used as the primary range control in this type of abort situation. The results of Fig. 12a show, however, that the guidance system is capable of utilizing almost full vehicle capability.

Two angles were considered in the re-entries shown in Fig. 12b for a velocity of 32,000 fps. At this velocity the vehicle has the capability, at the angle of $-4.3°$, of extremely long range. As shown, however, the guidance system is

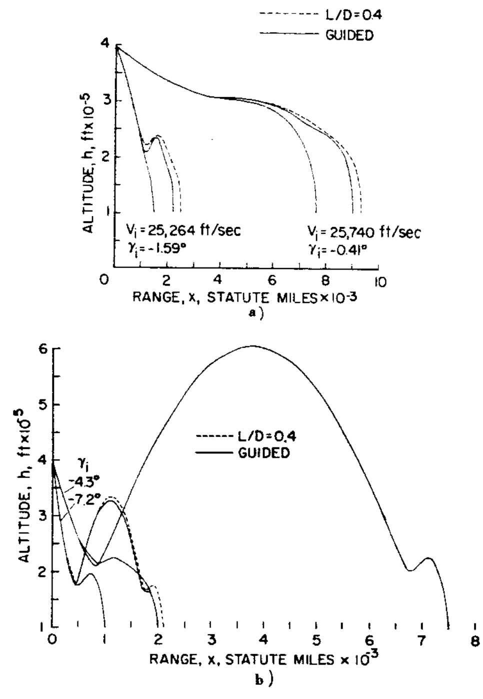


Fig. 12 Abort conditions: a) re-entry from circular orbit, b) re-entry at 32,000 fps.

incapable of achieving a range greater than 7500 miles for this off-design condition. At the steeper entry angle of $-7.2°$ the guidance is again able to utilize almost full vehicle capability.

The density deviation from the 1959 ARDC atmosphere used in this study varied linearly from zero at 100,000 ft to ±50% at 400,000 ft altitude. The L/D variations considered were either 5% greater or less than the value given by Eq. (14). A summary of these effects on guidance capability at various ranges is presented in Fig. 13. The solid line is a repeat of the information given in Fig. 11, that is, the guidance capability under nominal conditions. It can be seen that the L/D variations affected the capability relatively little. At long ranges, however, the density variation caused a considerable loss in the guidance capability. It is anticipated that including a component in the control equation sensitive to density deviations (the adaptive feature of Ref. 5) will make a marked improvement. This will be investigated in the near future.

Concluding Remarks

In this study a modified perturbation theory has been applied to the problems of lunar landing and earth re-entry guidance. It has been shown that if velocity is used as the independent variable in the guidance equation and if the linear theory gains are appropriately weighted, then one reference trajectory can be used successfully despite large

Fig. 11 Guidance capability.

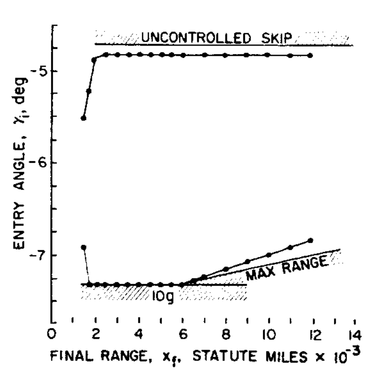
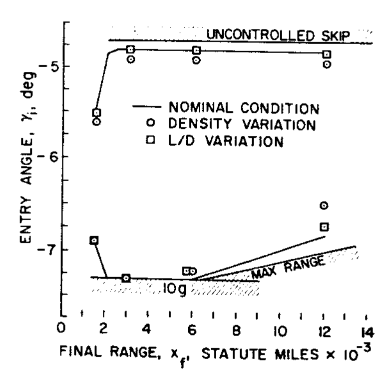


Fig. 13 Effect of off-design conditions on the guidance capability.



errors in nominal or initial conditions. The use of a single reference trajectory in each problem means that the guidance method requires little storage capacity.

In the lunar landing study, the guidance capability for a control system formulated with the gravity turn as the reference trajectory was far superior to one formulated with a constant-pitch-rate reference trajectory. With a single gravity turn reference trajectory, the guidance system could compensate for initial range errors of 100% of the reference value with a small additional fuel increment, equivalent to a characteristic velocity of 70 fps.

In the earth re-entry problem it was found that with a single reference trajectory it was possible to obtain a guidance capability from 1500 to 12,000 miles for a range of entry conditions which utilized virtually all of the vehicle's capability.

For the abort conditions considered in this paper, the guidance system was generally able to make almost full use of the vehicle's range capability.

Errors of 5% in vehicle L/D had little effect on the capability of this guidance scheme.

Density variations from the nominal affected the long-range guidance but had little effect on guidance capability for ranges less than 6000 miles.

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